The Physics of the Familiar

How paint dries, the way flags flutter, how Nature discovered origami, and other marvels of the physical world

by JONATHAN SHAW

"Just because something is familiar doesn't mean you understand it. That is the common fallacy that all adults make—and no child ever does," says Lakshminarayanan Mahadevan, England de Valpine professor of applied mathematics. Mahadevan enjoys explaining mathematically the phenomena of everyday life: practicing the old-fashioned method of scientific inquiry called natural philosophy, where one wonders about *everything*.

He has compared instability in nanotubes to the wrinkling of an elephant's trunk, and explained, together with his students and postdocs, how the leaves of the Venus flytrap can buckle shut fast

Photographs by Jim Harrison

enough to trap a fly.* With former postdoctoral fellow Enrique Cerda, now a physics professor in Chile, he has devised equations to describe how fabric drapes on a clothed figure or from a table. Inspiration is everywhere. "Look around the room," he tells his students. "Look out of the window." Mahadevan sees trees lifting water from roots to crown, or perhaps a drop of paint drying in the sun, and wants to understand in a deep sense what is happening right before his eyes.

W HAT WILL HAPPEN as the drop of paint dries? Why should we care?
"In the same way, you may ask, 'Why should you climb a mountain?'" Mahadevan says. "Some people do it for the challenge, but there is another aspect to it, and that is that you can actually see around you. You can see where you are." But the path straight up the mountain may be too hard. "The difficulty could be technical, or experimental, or it could be mathematical—it doesn't really matter." But then, he continues, you see a little hill, and think, "Maybe I'll climb up the hill. I will still be able to see around me—maybe not as far as from a mountain, but perhaps, once I get up there, I will find a path from that hill to a larger hill, and so on, from which I will be able to see the range, to understand the lay of the land, and know how to approach questions of a similar kind."

So with the paint drop. As it dries, a skin will form. That is the natural consequence of liquid solvent leaving the surface fastest. "Now you have a skin, which is covering a drop, which is sitting on a surface, and you want to remove more liquid from inside. That is like drying a raisin," says Mahadevan. When you remove some of the volume from beneath a fixed amount of surface skin, "the only thing the skin can do is wrinkle."

Paint often wrinkles—but sometimes it cracks. This seems odd, he points out, because in wrinkling, things are pushed together, whereas cracks form when things are pulled apart. What accounts for these antithetical behaviors?

The answer, he explains, depends on whether and how the drop is attached to the surface. "For example, if I have sliced the grape in half and stuck it down on a surface," then it can't shrink easily because it is stuck to the substrate, and it will rip. "On the other hand, if the drying grape is not attached," removing liquid from the inside will cause it to wrinkle as it shrinks.

Changing one little parameter changes the outcome. Using this knowledge, Mahadevan points out, "You could learn to engineer materials cleverly." The roughness imparted by wrinkles might prevent things from getting stuck to a surface, so that it becomes self-cleaning (like a ridged Circulon skillet). Or a lifesaving drug could be hidden in a crack of material

*See "Leaves That Lunch," May-June 2005, page 14. The work was done together with his student Jan Skotheim (now a postdoctoral fellow at Rockefeller University) and postdocs Yoel Forterre (now an assistant professor of physics at Marseilles) and Jacques Dumais (now an assistant professor of biology at Harvard).

engineered so that, whenever it shrinks or swells, the drug is exposed.

But the applications are not Mahadevan's main focus. He is seeking a general understanding of how things are ordered and shaped in space and time, and strives for simple answers. "The simplest answers often are very powerful," he points out, "since they allow one to move from one problem to another, often via analogy." For example, "If you get an idea looking at a particular system on a particular scale," can it be used to explain other things that initially seem very different, but are not? "We have wrinkles on our skin," he points out, "but, on a grander scale, wrinkles on the earth are called mountains. We have made a little bit of progress explaining their shape and size by doing experiments with balloons, which are very, very naively like the earth's crust."

The crust moves, he explains; it floats on a hot mantle of molten iron and silicate rock. Along the mid-Atlantic ridge, the mantle wells up and new crustal material is formed; in other places the crust falls back into the mantle. "If you look at the globe, you have seen perhaps that subduction zones [where two tectonic plates meet and one is forced beneath the other] are never, or very rarely, straight lines." Think of the islands of Japan or the Aleutians off the coast of Alaska. "They are almost always along arcs," he notes. "Why? What sets the size of the arc? And why is the arc cuspate—not one uniform arc, but many little arcs?"

Mahadevan suggests an answer based on very simple ideas. Newly formed hot crust is buoyant, which is why it rises. As it surfaces, it begins to cool and becomes heavier, even as it continues to be pushed up by new molten material beneath. Eventually, however, the newly formed crust's weight causes it to fall back. "That much is known," says Mahadevan. "The question is, does it fall down smoothly, gently, or does it fall down suddenly? If suddenly, does it fall along a straight line or does it fall along an arc? And very, very naively," he says, answering his own question, "it *must* fall along an arc." As the hardened rim of the crust collapses, it is moving toward a smaller diameter, which

means the perimeter has to shrink. But "the perimeter *can't* shrink very quickly, so instead, what will happen is, it will buckle." He is currently working with postdoctoral fellow Haiyi Liang and geophysicist Rebecca Bendick of the University of Montana to test some of these ideas using real geological data. The work builds on subjects Mahadevan has studied in the past, like wrinkles in human skin—or on the surface of a drop of paint.

The goal of Mahadevan's science is to get at the essence of complex phenomena in the everyday world with explanations that anyone can understand, but that can also be described with full-blown mathematics. "Maha is not only a tremendous applied mathematician, he truly is a kind of Renaissance thinker," says the dean of the School of Engineering and Applied Sciences, Venkatesh Narayanamurti. "Is he an applied mathematician? Is he an engineer? Is he a computational biologist? Or is he an applied physicist? He is all of the above," Narayanamurti adds. "I hadn't realized how much that was so until I got to know him after he joined the faculty. We are very fortunate in the last seven or eight years to have been able to recruit people like him: top-flight theoreticians who are solving important intellectual problems and doing all kinds of applied work as well." As it happens, Mahadevan's students and postdoctoral fellows, past and present, work in a variety of departments and disciplines, including biology, chemistry, engineering, mathematics and physics; at Harvard, his former postdoc Jacques Dumais is an assistant professor of organismic and evolutionary biology (OEB), and former student Adam Cohen is an assistant professor of chemistry.

TRAINED INITIALLY as an engineer at the Indian Institute of Technology-Chennai, Lakshminarayanan Mahadevan (*la-shmin-ah-*RYE*-ah-non ma-ha-*DAY*-vun*) turned to applied mathematics and mechanics under the tutelage of mathematician Joseph Keller (now professor emeritus) at Stanford, where he earned his Ph.D. in 1995; he was then briefly a postdoctoral fellow in physics and math at the University of Chicago. He held professorships in mechanical engineering at MIT, and in ap-

plied mathematics and theoretical physics at Cambridge University, before arriving in 2003 at Harvard, where he also has appointments in OEB within the Faculty of Arts and Sciences and in systems biology at the Medical School (HMS). In addition, he is a visiting professor of mathematics at Oxford University, and of biology at the National Center for Biological Sciences in Bangalore.

Although the problems he tackles vary in complexity, none is trivial in Mahadevan's eyes. He does not believe in a hierarchy of problems, or even that their solutions belong to particular disciplines. "A problem is just that: a problem," he says. "Nature does not tell us what kind of a problem it is—a physics problem, a biology problem, an engineering problem, an important problem, an unimportant problem... Nature couldn't care less." For

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example, he recently helped explain a problem puzzling the laboratory of a colleague, Mallinckrodt professor of physics and of applied physics David A. Weitz: why, in one experiment, a thin film of mud between two solid surfaces was cracking sporadically as it dried. The crack would remain still for a while, Mahadevan says, then "zip through suddenly and stop, pause for a while, then zip through and stop again." Mahadevan helped explain what was going on as the mud dried, using very simple ideas about the way water moves through soft materials.

Then, aware of the deep mathematical analogy between the rate at which materials dry and the rate at which they cool, he realized that his work on mud could be transposed to another experiment being done by two Canadian researchers, Lucas Goehring and Stephen W. Morris, who were seeking to explain the formation of giant, jointed basalt columns found in such places as the Devil's Postpile National Monument in east central California. Often perfectly six- or eight-sided, the columns present in profile the appearance of enormous hitching posts; they were
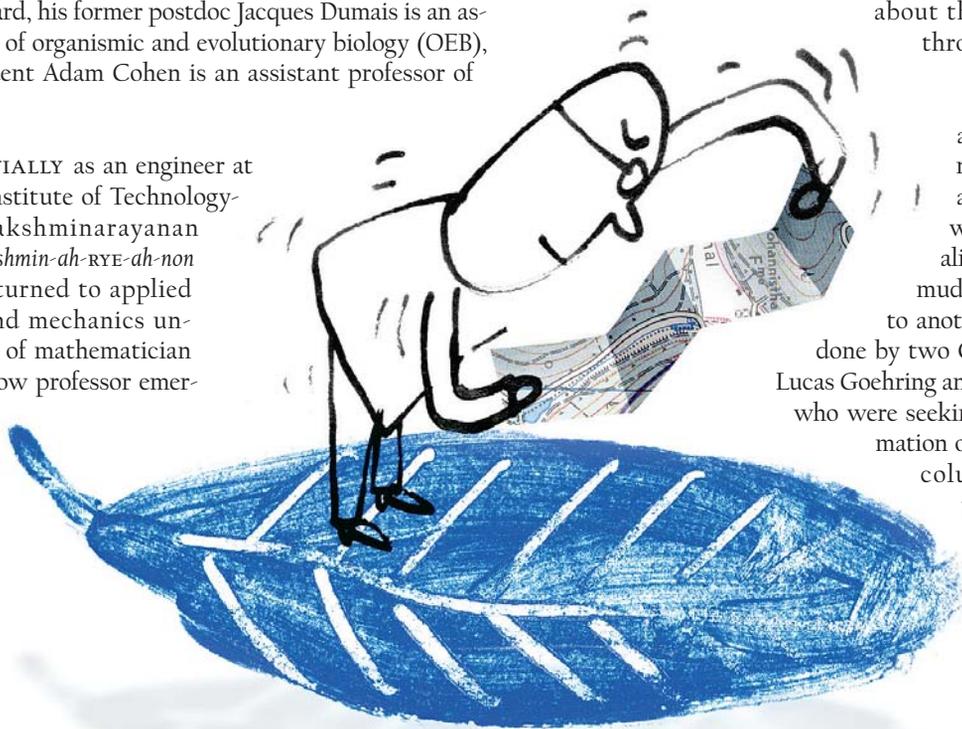
formed when volcanic rock cracked as it slowly cooled.

Mahadevan, who grew up in India, tells a traditional story about Krishna where mud becomes metaphor. "In Hindu mythology, Krishna is divine," he begins. "However, because there was a prophecy that he would overthrow an evil king, his origins when he was a baby were hidden from almost everybody. So when Krishna was born, his mother surreptitiously sent him away to be brought up by a foster mother who didn't know who he was. As in all mythologies, there were premonitions [of greatness], but growing up with his foster mother, he would go out like all children and play in the mud. One day he started to eat the mud, putting it in his mouth. And his [foster] mother, from afar, said, 'Don't do it.'" Krishna kept eating the mud. "Again [she] said, 'Don't do it,' and yet he continued. So she came up to him, and when she opened his mouth to take out the mud, she looked—and she saw the universe.

"Without ever claiming all the grandeur that the story actually suggests," says Mahadevan, "you just have to look and you will find interesting things everywhere."

Mahadevan has a particular knack for choosing interesting problems—and for coming up with solutions that are "as simple as possible, but no simpler," says his doctoral student Madhav Mani, echoing Einstein. "When he looks at a problem, he is able to strip away all the junk and say, 'This is the essence of the phenomenon we are looking at.'" It is a gift Mahadevan shares with students, from first-years in his freshman seminar, "Science of Everyday Life," to graduate students in Engineering Sciences 216, "Biological Dynamics," to his half-dozen most advanced postdoctoral fellows. "[Applied mathematicians] are all technically as good as each other," says Mani, "but the ability to see, for example, that there is a competition between two areas of physics leading into one problem," either one of which might offer an answer, and to focus on the one that holds the key to the solution: "this is the kind of thing that you want to learn as a Ph.D. student."
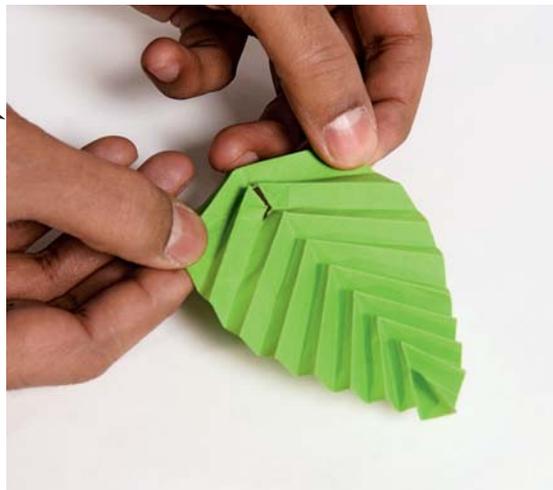
Using an origami model, Mahadevan demonstrates how folds in the leaves of a hornbeam or a beech are coupled, allowing them to easily open and close.

IN HIS OFFICE, Mahadevan keeps an origami model of a leaf made of green paper, patterned like the leaves of beech or a hornbeam. "This is a very clever design," he says admiringly. "You can open and close the leaf by playing with just one place." Mahadevan manipulates the model with his fingers. A plant would use water pressure to do the same thing. "The folds in this leaf" he says, "are very different from those in a map, in which each fold is decoupled," meaning that every fold can go either way—which is what makes road maps hard to close. In the leaf, on the other hand, all the folds are coupled to each other, so they open and close in unison. This same pattern, known as Miura-ori (after the Japanese engineer Koryo Miura), appears again and again in nature—in flowers and insect wings, for example, which have nothing in common at the genetic or molecular level—an apparent case of convergent design. Mahadevan thinks he knows why.

Looking at a "serendipitous experiment" associated with the wrinkling of gelatin, he found that this Miura-ori pattern arises naturally as the most efficient solution to a physical problem. When gelatin shrinks in two directions, the pattern develops spontaneously. "The analogue in biology would be that cells are shrinking, or expanding or reproducing, or dying" at a greater rate on a surface layer of a tissue than on an inner layer, forcing the cells to push together. "That is all you need," says Mahadevan. "You don't need to control where this fold or that fold will occur. The rest just follows, following the laws of physics, nothing more complicated than that"—a plausible explanation for why these patterns appear in so many places.

The underlying principle, which involves the differential expansion or contraction of cells, can also produce mechanical effects. "If you wet a pine cone, it will close," Mahadevan points out. "If you dry it, it will open." This occurs when water swells one part of the pine cone more than another, as in a bimetal thermostat where two strips, made of different metals that expand at differ-
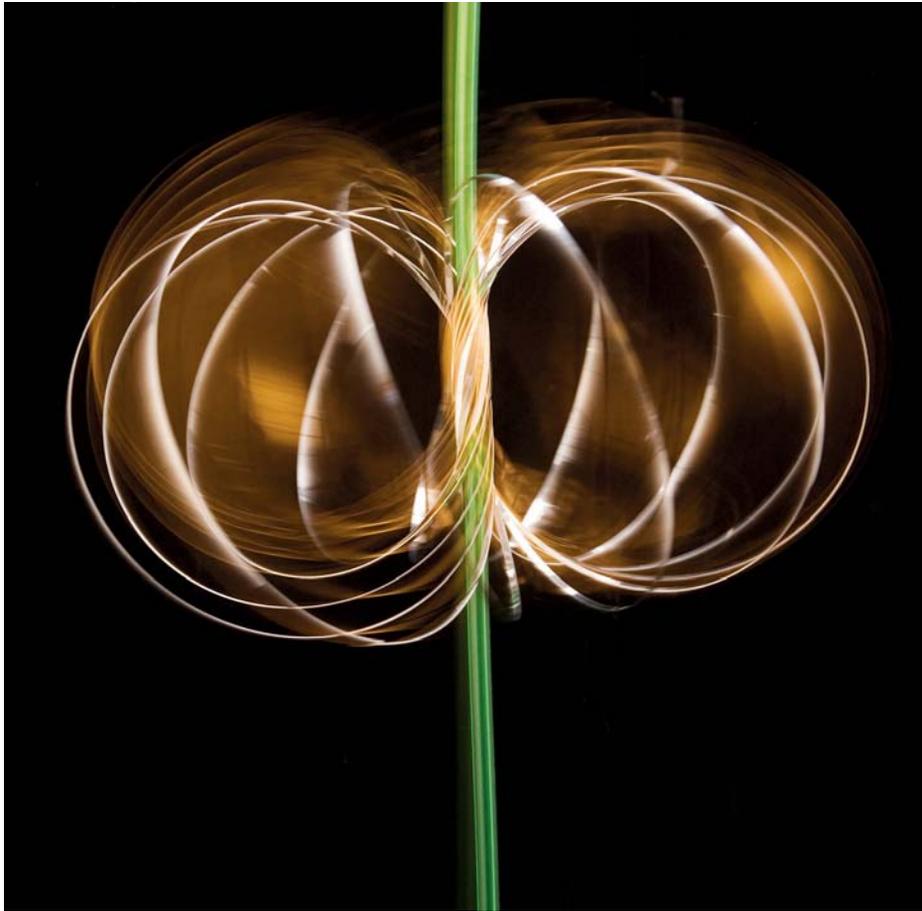
One of the "toys" in Mahadevan's office forms a toroidal bubble—a doughnut—as interlocked metal rings spin down around a central tube. He hopes to learn what limits the speed of its descent.

try, but also to think about the folding of the brain during embryogenesis, when gray matter packs itself into the tight confines of a skull.

Crochet, he explains, is made by knotting thread or yarn. "Suppose I wanted this object to be a flat, circular disk," he says. "Starting at the center, I would increase the number of knots linearly with the radius," to make, in effect a round hot pad. If, instead, as he worked out from the center, he decreased the number of knots at the perimeter, a spherical cap would start to form. But when the number of knots at the perimeter increases exponentially, you begin to get something like Daina Taimina's creation. The exponential growth at the edges leads to excess material that can be accommodated only by having folds. "This is a very deep mathematical result that goes back, in fact, to David Hilbert and Henri Poincaré," Mahadevan explains. He is interested in what happens next. "Geometers and mathematicians will tell you that as the structure grows more and more it will self-intersect, because there is too much material, and the only way to accommodate it all is basically to have it go through itself. Of course, this cannot happen with real material. Instead, the sheet forms complex folds."

In the case of the brain, for example, "you have a flat sheet that is growing rapidly" inside an enclosed structure, the skull. "Brain folding is a difficult problem, which a number of research groups are working on," Mahadevan says. Rather than tackle it directly,

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Visit harvardmag.com/extras to watch Mahadevan create wrinkles, folds, and a flutter.

ent rates in response to temperature, have been stuck together. When the metal on one side swells, the thermostat bends away to the opposite side to accommodate the expansion. When the metal contracts, the apparatus swings back the other way. A pine cone with seeds generally stays closed when it is on the tree, but as soon as it falls, it dries and opens, and the seeds start to disperse. "These are solutions to problems in biology, from which we may learn something once in while."

T HE OFFICE ALSO INCLUDES a collection of toys and interesting objects that once amused his children on weekends when they accompanied him to work. There is a purple cape, purloined from his daughter, that drapes and folds dramatically; a model of a biological structure that has kinks in it and can rotate; tubes for making mathematical models; and interlocked metal rings that form a toroidal bubble—one shaped like a donut—when they spin around a piece of flexible tubing that loops through the donut's center hole. Frequently, the objects illustrate ideas he has been working on. "This is a crochet made by a mathematician at Cornell, Daina Taimina," he says, holding up an object that resembles a bell with a wavy lower edge, or more fancifully, a flamenco dancer's frilled dress. He uses it to illustrate a fundamental question in geome-

"There is a unity to these ideas. Science is a continuum, and the goal should be to explain the largest number of observations with the smallest number of assumptions."

he has been climbing a smaller hill: he has been studying the growth of seaweed.

Kelp, the blades of which can grow as long as 10 meters, lives in shallow ocean waters. Typically, these algal blades are flat in the center and ruffled at the edges. But the amount of ruffling depends on the environment, says Mahadevan, whose interest in the problem was sparked by University of California, Berkeley biologist Mimi Koehl while he was on sabbatical there. If moved

into fast-flowing water, the plant loses its ruffles. Transplant the same algal blade back to areas with gentle currents, and the ruffles regrow. Researchers think that the advantage to the plant is to ensure that it wiggles—exposing as much of its surface as possible to the sun, maximizing photosynthetic activity—but doesn't flap so much that it breaks in strong currents. "Of course the plant is not a sentient being that knows when to grow or not," Mahadevan explains. "The *real* question is, how does stress connect to growth?"

STRUCTURE IS A COMMON THEME in Mahadevan's work: how shape arises and how things change shape and flow. "How does structure essentially force certain things to happen," he asks, "predetermining or very strongly constraining function in physics, chemistry, engineering, mathematics, geometry, biology, and medicine?"

Thus Mahadevan's interest in the way plants move water has led to one collaboration exploring how they grow pollen tubes and other structures using hydrostatic pressure, and another exploring how the design of veins in a plant's leaves work to control evaporation in different climates. Working in his lab with John Higgins, a clinical fellow in pathology from HMS, and with his experimentalist colleague Sangeeta Bhatia, who is at MIT, he is studying the physical basis for sickle cell disease—in which hemoglobin sometimes forms a flexible rod inside the balloon-like red blood cells, causing cells to take on the characteristic sickle shape that can block capillaries—using a small device that mimics symptoms of the disease. The goal is to understand the mechanisms associated with flow blockage, while also developing technologies that might be useful for individualized medical testing and for testing potential

The same rules of geometry and physics can govern wrinkling in objects as small as carbon nanotubes and as large as an elephant's trunk. Mahadevan makes the point with a tube of rolled rubber. (A video demonstration may be viewed on line.)
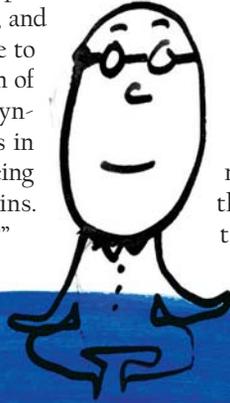
drugs without using human subjects. And based on experiments in the HMS laboratory of Sabbagh professor of systems biology Timothy Mitchison, he has come up with a new way of thinking about the inside of a cell. "Is it a fluid? Is it a solid? My theory, which has directly testable predictions, is that it is neither, but rather a fluid-infiltrated sponge." Mahadevan knows that such wide-ranging interests leave him exposed to the charge of dilettantism. To which he replies, "I hope that I am not a butterfly, but more like a bee. A bee pollinates. I move from flower to flower. I hope, once in a while, I am successful."

The kelp is an example of a recurrent theme among Mahadevan's interests: how growth leads to form, and how form changes function in the context—in this case—of having more or less light. He has developed a mathematical model of cellular growth in an algal blade, which shows that the pattern of ruffling arises naturally when the rims of a blade grow faster than the interior. The rims form a series of saddles, similar to the waves in the crochet, in order to accommodate their greater length. "The same thing happens when I take sheets of paper and wet them and dry them and different parts wet and dry differently," Mahadevan says. "Spray water on a paperback in the bathroom and when it dries it is very difficult to keep it shut again.... The point is that there is a unity to these ideas. Science is a continuum, and the goal should be to explain the largest number of observations with the smallest number of assumptions."

Mahadevan moves easily from modeling the way kelp grows to a simple understanding of how it sways in a current, based on his work examining how flags flutter. He holds a thin sheet of paper to his lips, and blows softly. The paper gently rises. Then he blows rapidly, and the sheet begins to flutter wildly. "There are many prob-

A wind blowing would push his arm first up, then down, as it overshot a position parallel to the direction of the wind. In this way, a flow can be converted to an oscillation, "something that has been understood since the time of Lord Kelvin." But of course a flag is not rigid. "Even in the absence of a fluid, if you hit the flag it will deform and then come back to rest." The combination of this *elastic* response and the fluid-driven response led to "a precise description of how you can push energy from the fluid into the solid and make it vibrate," he says. And of course one can turn the problem around, turning AC movements into DC locomotion. "Think of an eel that [propels itself forward] using waves of undulation that propagate along its body." In a recent paper with his student Zengcai Guo, Mahadevan described how snakes "swim" this way on land, by "pushing off against protuberances." And more whimsically, motivated by watching skates and rays skimming along the bottom of an aquarium, he has calculated, with student Jan Skotheim and Argentina, the conditions under which undulating carpets—actually thin films—could be made to fly.

Mahadevan is a testament to science driven primarily by curiosity. "Nobody questions whether art or music is useful and important and relevant," he says. In the same way, he believes, there ought to be room for science pursued simply because it is part of human culture. As a teacher, he aspires to whet and satisfy the natural scientific curiosity that drives such inquiry. No degree in mathematics is required. "To be able to experience some of the wonder and the mystery," he says, "you just have to probe the immediate environment around you." ▽

lems where you move from a steady behavior to an oscillation," he observes. In the 1930s, as planes began to fly faster and wings sometime broke off as a result of high-speed vibrations, aeronautics engineers devised ways to design around the problem. Similar wind-driven oscillations famously brought down the Tacoma Narrows Bridge in 1940. Mahadevan and a postdoctoral fellow, Médéric Argentina, now an assistant professor in France, realized the flutter of a flag could be used to unify a whole class of problems in a single conceptual idea: that a fluid such as air or water can actually couple different modes of energy.

"What a flag does," says Mahadevan, "is essentially convert a uniform DC [direct current] flow to AC [alternating current], if you think of it in terms of electrical analogies." The steady flow of the wind is converted into the oscillations of the flag. The simplest way to think of this is to imagine that the flag is rigid. Mahadevan raises his arm to demonstrate, using his elbow as a pivot point.

A purple cloak demonstrates—dramatically—how fabric drapes and folds.

Jonathan Shaw '89 is managing editor of this magazine. Readers interested in additional information on L. Mahadevan's work may wish to visit his website, www.seas.harvard.edu/softmat.